

Nuclear Structure-Dependent Radiative Corrections to the Hydrogen Hyperfine Splitting

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Abstract

Radiative corrections to the Zemach contribution of the hydrogen hyperfine splitting are calculated. Their contributions amount to $-0.63(3) ppm$ to the HFS. The radiative recoil corrections are estimated to be $0.09(3) ppm$ and heavy particle vacuum polarization shifts the HFS by $0.10(2) ppm$. The status of the nuclear-dependent contributions are considered. From the comparison of theory and experiment the proton polarizability contribution of $3.5(9) ppm$ is found. The nuclear structure-dependent corrections to the difference $\nu_{hfs}(1s) - n^3\nu_{hfs}(ns)$ are also obtained.

1 Introduction

The hyperfine splitting of the ground state of the hydrogen atom is one of the most precise measured values [1, 2]

$$\nu_{HFS}(1s) = 1420405.7517667(9) \text{ kHz}, \quad (1)$$

but the theory is not able to obtain this result with such an accuracy. The main problem is due to the proton structure. This work is devoted to nuclear structure-dependent contributions.

An expression of the leading nuclear structure-dependent correction to the hyperfine splitting in the hydrogen atom was found by Zemach [3]. Later nuclear structure-dependent corrections were investigated in Refs. [4, 5, 6, 7, 8], but the radiative structure-dependent corrections have not yet been found. The pure radiative corrections are evaluated in the next section. The third section is devoted to radiative recoil corrections. After that the vacuum polarization of heavy particle is discussed. The last sections consider the status of the nuclear structure-dependent terms.

2 External Field Approximation

2.1 Leading structure-dependent term

The Zemach expression of the leading nuclear structure-dependent correction to the ground state hyperfine splitting in the hydrogen atom has the form [3]

$$\Delta\nu(Zemach) = \nu_F \left(-2Z\alpha m_e \int \int d^3\mathbf{r} d^3\mathbf{r}' \rho_E(\mathbf{r}) \rho_M(\mathbf{r}') |\mathbf{r} - \mathbf{r}'| \right),$$

or

$$\Delta\nu(Zemach) = \nu_F \frac{2Z\alpha m_e}{\pi^2} \int \frac{d^3\mathbf{p}}{\mathbf{p}^4} \left[\frac{G_E(\mathbf{p}^2) G_M(\mathbf{p}^2)}{1 + \kappa} - 1 \right] \quad (2)$$

where $\rho_E(\mathbf{r})$ and $\rho_M(\mathbf{r})$ are the proton electric charge and magnetic moment distribution respectively, $G_{E/M}(\mathbf{p}^2)$ is the Sachs electric/magnetic form factor, κ is the proton anomalous magnetic moment. The Fermi energy ν_F is defined as the ground state hyperfine splitting in the nonrelativistic theory. Here, relativistic units in which $\hbar = c = 1$ and $\alpha = e^2$ are used. Z is the

nuclear charge in units of the proton charge. It is equal to one in the hydrogen atom, but some results in this paper like eq.(2) are valid for any low- Z hydrogen-like atom.

2.2 Dipole approximation

In the well-known dipole approximation of the electromagnetic form factors of the proton

$$G_E(\mathbf{p}^2) = \frac{G_M(\mathbf{p}^2)}{1 + \kappa} = \left[\frac{\Lambda^2}{\Lambda^2 + \mathbf{p}^2} \right]^2 \quad (3)$$

it is easy to solve eq.(2) [8]:

$$\Delta\nu(Zemach) = \nu_F \left(-\frac{35}{4} \frac{Z\alpha m_e}{\Lambda} \right). \quad (4)$$

2.3 Vacuum polarization

To calculate the contribution of the electronic vacuum polarization one can insert the well-known asymptotic behaviour of the polarization operator

$$2 \frac{\alpha}{\pi} \left\{ \frac{1}{3} \log \frac{\mathbf{p}^2}{m_e^2} - \frac{5}{9} \right\} \quad (5)$$

into the right hand side of eq.(2). Integrating eq.(2) within the dipole approximation(eq.(3)) yields:

$$\Delta\nu(structure - VP) = \Delta\nu(Zemach) \cdot \frac{\alpha}{\pi} \left\{ \frac{2}{3} \log \frac{\Lambda^2}{m_e^2} - \frac{634}{315} \right\}.$$

This result is in agreement with an estimate found in Ref. [8].

The logarithmic part of this result may be used for any low- Z hydrogen-like atom (if the magnetic square radius (R_M) is approximately equal to the charge square radius (R_E)) after substituting $\Lambda \rightarrow \sqrt{12}/R_E$.

2.4 Self energy

The self-energy contribution is evaluated by using an explicit asymptotic expression of the one-loop insertion into the electron line. The result is

$$\Delta\nu(\textit{structure} - e\textit{-line}) = \Delta\nu(Zemach) \cdot \frac{\alpha}{\pi} \left\{ -\frac{5}{4} \right\}. \quad (6)$$

The coefficient $(-5/4)$ arises from nontrivial radiative insertion into the electron line $(-7/4)$ [9] and from the anomalous magnetic moment contribution $(1/2)$. It should be mentioned that this result has been obtained without the use of the dipole fit (eq.(3)) and it is also valid for any low- Z hydrogen-like atom. It can be used in a wide interval of the nuclear charge up to $Z = 25$ and the uncertainty is expected as to grow $(\pi Z\alpha)^2$ in relative units.

2.5 Binding corrections

The higher order binding corrections have the order $(Z\alpha)^2$ or $(Z\alpha m_e)/\Lambda$ in units of $\Delta\nu(Zemach)$. They are small and it is enough to take into account only their logarithmic parts. The $(Z\alpha)^2$ -term arises from the Dirac correction to the wave function

$$\Delta\nu(\textit{structure} - Dirac) = \Delta\nu(Zemach) \cdot \frac{(Z\alpha)^2}{2} \log \frac{1}{(Z\alpha)^2}.$$

The $(Z\alpha m_e)/\Lambda$ -contribution IS DUE TO the nuclear charge distribution and the Fermi interaction. The result is

$$\Delta\nu(\textit{structure} - charge) = \Delta\nu_F \cdot \left(-\frac{2}{3} (Z\alpha m_e)^2 r_p^2 \log \frac{1}{(Z\alpha)^2} \right).$$

These corrections are small, but they are considered here, because only the binding corrections can contribute to the hfs splitting of the higher- l state or to the difference

$$\Delta_{hfs}(n) = \nu_{hfs}(1s) - n^3 \nu_{hfs}(ns).$$

The result for the difference can be found in the non-relativistic approximation (cf. Ref. [14])

$$\delta\Delta_{hfs}(n) = \Delta\nu(Zemach) \cdot (Z\alpha)^2 \left[\psi(n+1) - \psi(2) - \log n - \frac{(n-1)(n+9)}{4n^2} \right], \quad (7)$$

where $\psi(z) = (d/dz) \log \Gamma(z)$, and for the hfs of states with $l > 0$

$$\delta\nu(nl_j) = \frac{\Delta\nu(Zemach)}{n^3} \cdot (Z\alpha)^2 \frac{n^2 - 1}{4n^2} \delta_{j,1/2} \delta_{l1}.$$

The $(Z\alpha m_e)/\Lambda$ -contribution may be found in the same technics useds in Ref. [15]

$$\delta\Delta_{hfs}(n) = \Delta\nu_F \cdot \left(-\frac{2}{3} (Z\alpha m_e)^2 r_p^2 \right) \left(\frac{n-1}{n} - \log n + \psi(n) - \psi(1) \right) \quad (8)$$

The binding corrections of eq.(7) and eq.(8) shift the difference $\Delta_{hfs}(2)$ by 10^{-9} kHz only.

3 Recoil Contributions

3.1 The leading term

The final result of the pure recoil corrections was found in Ref. [8] by numerical means using the dipole approximation. It includes large numerical cancelation between different terms. This cancelation can be understood analytically from the leading logarithmic term [4, 5] for atoms with a non-structured nucleus (terms VO , VV and κ^2 of Ref. [8])

$$\Delta\nu(rec) = -\nu_F \cdot \frac{3Z\alpha m_e}{\pi m_p} \log \frac{m_p}{m_e} \cdot \frac{2(1+\kappa) - (1+\kappa)^2 + \frac{3}{4}\kappa^2}{1+\kappa}. \quad (9)$$

The coefficient

$$\frac{2(1+\kappa) - (1+\kappa)^2 + \frac{3}{4}\kappa^2}{1+\kappa}$$

is equal to 1 in muonium and 0.070... in the hydrogen atom.

In the following subsections radiative recoil corrections (i. e. radiative corrections to eq.(9)) are considered.

3.2 Electronic vacuum polarization

The electronic vacuum polarization term contains the same structure of the Dirac matrix as the leading term of eq.(9) and the same cancelation occurs. The estimate

$$\Delta\nu(VP - log) = -\nu_F \cdot \frac{2\alpha(Z\alpha)}{\pi^2} \frac{m_e}{m_p} \log^2 \frac{\Lambda}{2m_e} \\ \times \frac{2(1 + \kappa) - (1 + \kappa)^2 + \frac{3}{4}\kappa^2}{1 + \kappa}.$$

can be obtained by using the asymptotics of eq.(5) (see also [8]).

3.3 Self energy

The self-energy recoil contribution can be estimated from this contribution in the muonium atom, which includes a non-relativistic pole ("the δ' -term" in the definitions used in Ref. [10] or "the NR -contribution" in Refs. [11, 12]), logarithmic and constant contributions of two different structures of the Dirac matrix [10, 11, 12]. The numerically important contributions arise from the pole and logarithmic terms. They can easily be adjusted to the hydrogen atom. The constant can be used to estimate uncertainty. The anomalous magnetic moment contribution has to be added as well. The final numerical result is presented in the last section.

4 Heavy Particle Vacuum Polarization

The correction due to muonic and hadronic vacuum polarization have been treated for a point-like nucleus in Ref. [16]¹. These corrections are found here in the external field approximation (for details see Ref. [16]) with the dipole form factors of eq.(3). The numerical results are presented in the last section. The results for the point-like nucleus

¹That work contains some misprints: the left part of eq. (9) should be multiplied by 2π ; the result for the hadronic contribution to the HFS in eq.(10) and in the Table should be multiplied by 2.

$$\Delta\nu(\mu - VP - point) = \frac{3}{4} \alpha(Z\alpha) \frac{m_e}{m_\mu} \nu_F,$$

and

$$\Delta\nu(hadr - VP - point) \simeq 0.7(3) \Delta\nu(\mu - VP - point)$$

and for the finite-size nucleus are different. The finite-size nucleus results are only some the 30% of the point-like nuclear corrections. The results for the finite-size nucleus are given in this work within the external field approximation. The uncertainties are estimated by the unknown recoil contributions (cf. Ref. [16]).

5 Parameterization of the Dipole Fit

Because no reliable self-consistent values of the Zemach correction are known numerical results can be obtained only after reconsideration this correction. The parameter Λ which is needed for this calculation is directly connected with the proton square charge radius

$$r_p = \frac{\sqrt{12}}{\Lambda}.$$

Comparison of recent experimental results with the theory (see e. g. Ref. [17]) favors the newer value [18]

$$r_p = 0.862(12) \text{ fm},$$

or

$$\Lambda = 0.845(12) \text{ m}_p. \tag{10}$$

However, as most of the work in this field was done more than 15 years ago, the older proton radius of Ref. [20]

$$r_p = 0.809(11) \text{ fm},$$

or

$$\Lambda = 0.898(13) m_p \quad (11)$$

was used. It is well-known that the form factor from the older radius and the value $\Lambda = 0.898(13) m_p$ is good as long as the momentum transfer is not too low. Hence, the first problem is to understand what momenta are important in the integration of eq.(2). In order to solve this problem high and low momenta have been separated in the integral

$$\Delta\nu(Zemach) = \nu_F \frac{8Z\alpha m_e}{\pi m_p} \times \left\{ m_p \int_0^Q \frac{dp}{p^2} \left[(G_D(\mathbf{p}^2))^2 - 1 \right] + m_p \int_Q^\infty \frac{dp}{p^2} \left[(G_D(\mathbf{p}^2))^2 - 1 \right] \right\}. \quad (12)$$

The low momenta asymptotics has the form

$$\left[(G_D(\mathbf{p}^2))^2 - 1 \right] \simeq -4 \frac{\mathbf{p}^2}{\Lambda^2}, \quad (13)$$

and the high momenta asymptotic behaviour is

$$\left[(G_D(\mathbf{p}^2))^2 - 1 \right] \simeq -1. \quad (14)$$

The results of integrations are presented in Fig.1.

The main radius-dependent contribution arises from the low momenta asymptotics $(-4\mathbf{p}^2/\Lambda^2)$, which should be directly connected to the radius. Our results for several Q (see also Fig. 2)

$$\Delta\nu(Zemach) = \begin{cases} -40.92(59) \cdot 10^{-6} \cdot \nu_F, & Q = 0.30 m_p, \\ -41.07(68) \cdot 10^{-6} \cdot \nu_F, & Q = 0.35 m_p, \\ -41.24(68) \cdot 10^{-6} \cdot \nu_F, & Q = 0.40 m_p, \end{cases} \quad (15)$$

are obtained as the sum of the $4\mathbf{p}^2/\Lambda^2$ -contribution from the lower momenta with $\Lambda = 0.845 m_p$ and the average value of the remaining terms with $\Lambda = 0.845 m_p$ and $\Lambda = 0.898 m_p$. The uncertainty is obtained from the sum of

the squares of the parameter-induced uncertainty in the $4\mathbf{p}^2/\Lambda^2$ term and half of the difference for the contribution of $G_D^2 - 1 - 4\mathbf{p}^2/\Lambda^2$ and of higher momenta with $\Lambda = 0.845 m_p$ and $\Lambda = 0.898 m_p$. The details of calculations are contained in the appendix.

The results for different values of Q are almost independent of Q . They should be compared with results for the point-like proton

$$\Delta\nu(Zemach) = \begin{cases} -41.15(58) \cdot 10^{-6} \cdot \nu_F, & \Lambda = 0.845(12) m_p, \\ & r_p = 0.862(12) fm, \\ -38.72(56) \cdot 10^{-6} \cdot \nu_F, & \Lambda = 0.898(13) m_p, \\ & r_p = 0.809(11) fm, \\ -40.43(43) \cdot 10^{-6} \cdot \nu_F, & \Lambda = 0.860(9) m_p, \\ & r_p = 0.847(9) fm. \end{cases} \quad (16)$$

The last proton radius value is the result presented in the recent work [21].

One can see that the results of our estimate in eq.(15) are close to the result of eq.(16) for $\Lambda = 0.845(12) m_p$, but the uncertainties are a little higher. This agreement of eq.(15) with eq.(16) for $\Lambda = 0.845(12) m_p$ is due to the use of this value in our calculation of the $4\mathbf{p}^2/\Lambda^2$ term. Within the interval between $0.3 m_p$ and $0.4 m_p$ of value of Q the approach described here is expected to yields the best results. The value

$$\Delta\nu(Zemach) = -41.07(75) \cdot 10^{-6} \cdot \nu_F \quad (17)$$

is used for futher numerical calculation. We expect that this result is more safe than any simple dipole fit values from eq.(16).

6 Conclusion

In one of the latest works [8], devoted to the ground state hyperfine splitting in the hydrogen atom, compararison of theory and experiment leads to the difference

$$\frac{\nu_{HFS}(exp) - \nu_{HFS}(theo)}{\nu_{HFS}(exp)} = (0.56 \pm 0.48) ppm. \quad (18)$$

The theoretical expression excludes the unknown proton polarizability so it may be estimated by the difference in eq.(18). The theoretical limitation for the proton polarizability contribution is [22, 23, 24]

$$|\delta(\textit{polarizability})| < 4 \textit{ ppm}. \quad (19)$$

The result of this work is

$$\frac{\nu_{HFS}(\textit{exp}) - \nu_{HFS}(\textit{theo})}{\nu_{HFS}(\textit{exp})} = (3.5 \pm 0.9) \textit{ ppm} \quad (20)$$

instead eq.(18). The changes of the theoretical values are presented in Tables 1–3. The older comparison of theory and experiment in eq.(18) implies that the polarizability contribution is much lower than the limitation of eq.(19). However, our comparison in eq.(20) leads to a result close to this limitation.

The hyperfine splitting of the hydrogen ground state is more sensitive to the proton structure value than the Lamb shift. We hope that investigation of the hfs will lead to a better understanding of the proton and to a more accurate calculation of the Lamb shift, the precision of which is limited by the proton radius.

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A Q-integrals for calculation of the Zemach correction

1. The integrals over different area are

$$I_{<}(Q, \Lambda) = m_p \int_0^Q \frac{dp}{\mathbf{p}^2} \left[\left(G_D(\mathbf{p}^2) \right)^2 - 1 \right]$$

$$= -\frac{35}{16} \frac{m_p}{\Lambda} \operatorname{arctg} \left(\frac{Q}{\Lambda} \right) - \frac{19}{16} \frac{m_p Q}{\Lambda^2 + Q^2} - \frac{11}{24} \frac{m_p Q \Lambda^2}{(\Lambda^2 + Q^2)^2} - \frac{1}{6} \frac{m_p Q \Lambda^4}{(\Lambda^2 + Q^2)^3}$$

and

$$I_{>}(Q, \Lambda) = m_p \int_Q^\infty \frac{dp}{\mathbf{p}^2} \left[\left(G_D(\mathbf{p}^2) \right)^2 - 1 \right]$$

$$= -\frac{35}{16} \frac{m_p}{\Lambda} \operatorname{arctg} \left(\frac{\Lambda}{Q} \right) + \frac{19}{16} \frac{m_p Q}{\Lambda^2 + Q^2} + \frac{11}{24} \frac{m_p Q \Lambda^2}{(\Lambda^2 + Q^2)^2} + \frac{1}{6} \frac{m_p Q \Lambda^4}{(\Lambda^2 + Q^2)^3}.$$

2. The contributions of asymptotics of eq.(13) and eq.(14) are easy to find:

$$A_{<}(Q, \Lambda) = -4 \frac{m_p Q}{\Lambda^2}$$

and

$$A_{>}(Q, \Lambda) = -\frac{m_p}{Q}.$$

The contributions of remaining low-momenta term is denoted by

$$R_{<}(Q, \Lambda) = I_{<}(Q, \Lambda) - A_{<}(Q, \Lambda).$$

ALL RESULTS WITH DIFFERENT Λ ARE PRESENTED IN FIG. 1 AS FUNCTIONS OF Q .

3. The following combinations are used as the result and the uncertainty

$$I(Q) = A_{<}(Q, 0.845 m_p) + \frac{R_{<}(Q, 0.845 m_p) + R_{<}(Q, 0.898 m_p)}{2}$$

$$+\frac{I_{>}(Q, 0.845\,m_p) + I_{>}(Q, 0.898\,m_p)}{2}$$

and

$$\delta I(Q) = \left\{ \left(\frac{8\,\delta\,\Lambda\,m_p\,Q}{\Lambda^3} \right)^2 \right.$$

$$\left. + \left(\frac{R_{<}(Q, 0.845\,m_p) - R_{<}(Q, 0.898\,m_p)}{2} \right)^2 + \left(\frac{I_{>}(Q, 0.845\,m_p) - I_{>}(Q, 0.898\,m_p)}{2} \right)^2 \right\}^{1/2}.$$

The function $I(Q)$ is presented in Fig. 2.

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List of captions to the tables and the figures

Table 1: Old and new values of some relative contributions to the theoretical hfs (in ppm).

^a The following new values have been used: $\alpha^{-1} = 137.0359994(6)$ from Ref. [31] and $m_p/m_e = 1836.152667(4)$ from Ref. [32].

^b The result of Refs. [33, 34] used in Ref. [8] is incorrect (for details see review [29] and Refs. [35, 36]).

^c This is actually a result of Ref. [8], but the older proton radius was used there. The uncertainty given there was tripled here.

Table 2: Contributions to the relative shift of the theoretical result for the hfs (in ppm).

Table 3: Contributions to the uncertainty of the theoretical result for the hfs (in ppm).

Fig 1: Contribution to the integral in eq.(12) for $\Lambda = 0.898 m_p$. $I_>$ – contribution from higher momenta, $A_<$ – contribution of asymptotics of eq.(13) from lower momenta and $R_<$ contribution of the remaining terms.

Fig 2: The Zemach correction as a function of Q .

Term	Old	New	Ref.
$\nu_F - \nu_{exp}$	-1102.15	-1102.28	^a
$\alpha(Z\alpha)^2$	1.90(4)	1.84(12)	^b
$\alpha^2(Z\alpha)$	—	0.09	[25, 26, 27]
<i>higher ord.</i>	—	0.01(2)	[28, 29]
<i>VP – Structure</i>	—	-0.74(1)	this work
<i>SE – Structure</i>	—	0.12	this work
<i>Binding – Structure</i>		-0.01	this work
<i>Zemach</i>	-38.72(56)	-41.07(75)	this work
<i>Recoil – Structure</i>	5.22(14)	5.22(42)	^c
<i>Recoil – VP – Structure</i>	—	-0.02	this work
<i>Recoil – SE – Structure</i>	—	0.11(2)	this work
$\mu - VP - Structure$	—	0.07(2)	this work
<i>Hadr – VP – Structure</i>	—	0.03(1)	this work
<i>Weak interaction</i>	—	-0.06	[30]

Table 1:

Term	Shift
<i>Constants (α , m_e/m_p)</i>	-0.13
<i>QED</i>	0.04
<i>Zemach</i>	-2.36
<i>Radiative – Structure</i>	-0.63
<i>RRC – Structure</i>	0.09
<i>Heavy – VP – Structure</i>	0.10
<i>Weak interaction</i>	-0.06
<i>Total</i>	-2.95

Table 2:

Term	Uncertainty
<i>QED</i>	0.12
<i>Zemach</i>	0.75
<i>Radiative – Structure</i>	0.03
<i>Recoil</i>	0.42
<i>RRC – Structure</i>	0.03
<i>Heavy – VP – Structure</i>	0.02
<i>Total</i>	0.87

Table 3:

Fig 1

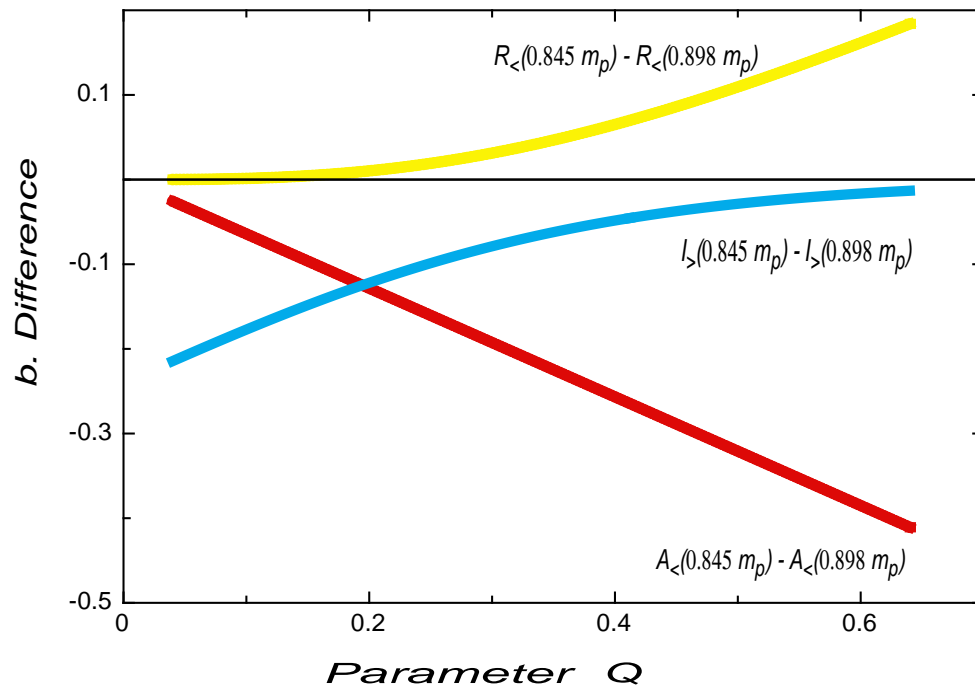
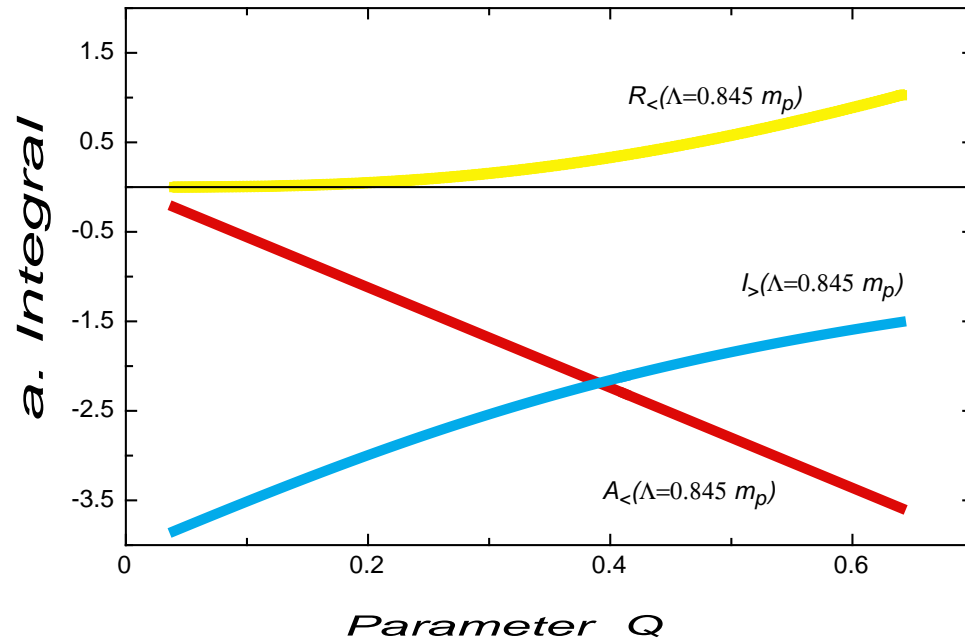


Fig. 2

